Indian Statistical Institute
Second Semester Back Paper Examination 2003-2004
B.Math (Hons.) II Year
Algebra IV

Time: 3 hrs Date: 23.07.2004

Note: Each question carries 10 marks.

1. For $t \in I\!R,\ f : I\!R \to \mathbb{C}$ continuous define $(\rho(t)f)(x) = f(x-t)$. Which of the following subspaces are $\rho(I\!R)$- invariant?

   a) the subspace of polynomials.
   b) the subspace of even function.
   c) the space of the functions $cosx, cox2x, ..., cosnx$.

2. a) For an irreducible character x of a finite group $G \neq \{1\}$, detrmine $\frac{1}{|G|} \sum\limits_{g \in G} x(g)$.

   b) Find the character of the representation of the group S_n on an n-dimensional vector space V with basis $\{e_1, \ldots e_n\}$ defined by $\sigma \cdot e_i = e_{\sigma(i)}$ for $\sigma \in S_n$.

3. Let V be the $\mathbb{C}$ vector space of homogeneous polynomials of degree n in 2 variables x, y. Consider the representation of $SU(2)$ on V by

$$\left(\rho\left(\begin{pmatrix} a & b \\ c & d \end{pmatrix}\right) f\right)(x,y) = f\left((x,y)\begin{pmatrix} a & b \\ c & d \end{pmatrix}\right)$$

   By looking at the restriction of ρ to the subgroup of diagonal matrices in $SU(2)$, prove that ρ is an irreducible representation of $SU(2)$

4. For a $n \times n$ matrix X, prove that

$$\left.\frac{de^{tx}}{dt}\right|_{t=0} = X$$

5. Find a homomorphism θ from $SU(2)$ to $SU(3)$ with kernel $=\{\pm I\}$.

6. Show that $L^2(S^1)$ decomposes into the Hilbert space direct sum of finite-dimensional irreducible representations of S^1.